OCU-PHYS-219
hep-th/0410132

$U(N)$ Gauge Model and Partial Breaking of $\mathcal{N}=2$ Supersymmetry [a]

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ABSTRACT

We briefly review a construction of $\mathcal{N}=2$ supersymmetric $U(N)$ gauge model in which rigid $\mathcal{N}=2$ supersymmetry is spontaneously broken to $\mathcal{N}=1$. This model generalizes the abelian model considered by Antoniadis, Patouche and Taylor. We discuss the conditions on the vacua of the model with partial supersymmetry breaking.

1. Introduction

Let us recall that partial breaking of extended rigid supersymmetries appears not possible on the basis of the positivity of the supersymmetry charge algebra:

$$\left\{\bar{Q}^i_{\alpha}, Q_{j\dot{\alpha}}\right\} = 2(\mathbf{1})_{\alpha\dot{\alpha}}\delta^i{}_j H. \tag{1}$$

In fact, if $Q_1|0\rangle = 0$, one concludes $H|0\rangle = 0$ and $Q_i|0\rangle = 0$ for all i. If $Q_1|0\rangle \neq 0$, then $H|0\rangle = E|0\rangle$ with $E > 0$ and $Q_i|0\rangle \neq 0$ for all i. The loophole to this argument is that the use of the local version of the charge algebra is more appropriate in spontaneously broken symmetries and the most general supercurrent algebra is

$$\left\{\bar{Q}^j_{\dot{\alpha}}, \mathcal{S}^m_{\alpha i}(x)\right\} = 2(\sigma^n)_{\alpha\dot{\alpha}}\delta^j{}_i T^m_n(x) + (\sigma^m)_{\alpha\dot{\alpha}} C^j_i, \tag{2}$$

where $\mathcal{S}^m_{\alpha i}$ and T^m_n are the supercurrents and the energy momentum tensor respectively. We have denoted by C^j_i a field independent constant matrix permitted by the constraints. This last term does not modify the supersymmetry algebra acting on the fields. The

[a] Talk given by H.I. at the 12th International Conference on Supersymmetry and Unification of Fundamental Interactions (SUSY2004), June 17-23, 2004, Tsukuba, Japan.

abelian model of [1] and our nonabelian generalization [2] provide a concrete example of this local algebra within linear realization from the point of view of the action principle.

2. $\mathcal{N}=2$ $U(N)$ Gauge Model

We consider a $U(N)$ gauge model which is composed of a set of $\mathcal{N}=1$ chiral superfields $\Phi = \Phi^a t_a$ and that of $\mathcal{N}=1$ vector superfield strengths $\mathcal{W} = \mathcal{W}^a t_a$ in the adjoint representation of $U(N)$. The t_a form $u(N)$ algebra $[t_a, t_b] = f_{ab}^c t_c$. The kinetic term for Φ is specified by the Kähler potential

$$K(A^a, A^{*a}) = \frac{i}{2}(A^a \mathcal{F}_a^* - A^{*a}\mathcal{F}_a), \quad \mathcal{F}_a \equiv \partial_a \mathcal{F}, \tag{3}$$

where $\mathcal{F}$ is an analytic function of A, and thus the Kähler metric takes the form $g_{ab^*} = \mathrm{Im}\,\mathcal{F}_{ab}$. The gauging of the $U(N)$ isometry is accomplished by following the method [3] and using the Killing potential

$$\mathfrak{D}_a = -\frac{1}{2}(\mathcal{F}_b f_{ac}^b A^{*c} + \mathcal{F}_b^* f_{ac}^b A^c). \tag{4}$$

For gauge fields we introduce the non-canonical kinetic action: $-\frac{i}{4}\int d^2\theta \tau_{ab}\mathcal{W}^a\mathcal{W}^b + c.c.$, where τ_{ab} is an analytic function of Φ. In addition, we include the superpotential $W(\Phi)$ and the Fayet-Iliopoulos D-term $\sqrt{2}\xi D^0$. The index 0 refers to the overall $u(1)$.

We found that our action is invariant under the $\mathfrak{R}$ action

$$\begin{aligned} \begin{pmatrix} \lambda^a \\ \psi^a \end{pmatrix} \to \begin{pmatrix} \psi^a \\ -\lambda^a \end{pmatrix} \quad &, \quad D^c + \frac{1}{2}g^{cd}\mathfrak{D}_d \to -(D^c + \frac{1}{2}g^{cd}\mathfrak{D}_d)\ , \\ \xi \to -\xi \quad &, \quad F^a + g^{ac^*}\partial_{c^*}W^* \to F^{*b} + g^{db^*}\partial_d W\ , \end{aligned} \tag{5}$$

if we choose τ_{ab} and $W(\Phi)$ as

$$\tau_{ab} = \mathcal{F}_{ab}, \quad W = eA^0 + m\mathcal{F}_0, \tag{6}$$

where e and m are real constants. The $\mathfrak{R}$ action is a discrete element of the $SU(2)$ R-symmetry that acts as an automorphism of $\mathcal{N}=2$ supersymmetry. Because our action is invariant under the $\mathfrak{R}$ action as well as the $\mathcal{N}=1$ supersymmetry transformation δ_{η_1}, it is invariant under the second supersymmetry $\delta_{\eta_2} = \mathfrak{R}\delta_{\eta_1}\mathfrak{R}^{-1}$. Thus, our action is invariant under the $\mathcal{N}=2$ supersymmetry.

As a result, the action of our $\mathcal{N}=2$ $U(N)$ gauge model is

$$
\begin{aligned}
\mathcal{L} \;=\; & -g_{ab^*}\mathcal{D}_m A^a \mathcal{D}^m A^{*b} - \frac{1}{4}g_{ab}v^a_{mn}v^{bmn} - \frac{1}{8}\mathrm{Re}(\mathcal{F}_{ab})\epsilon^{mnpq}v^a_{mn}v^b_{pq} \qquad (7)\\
& -\frac{1}{2}\mathcal{F}_{ab}\lambda^a\sigma^m\mathcal{D}_m\bar{\lambda}^b - \frac{1}{2}\mathcal{F}^*_{ab}\mathcal{D}_m\lambda^a\sigma^m\bar{\lambda}^b - \frac{1}{2}\mathcal{F}_{ab}\psi^a\sigma^m\mathcal{D}_m\bar{\psi}^b - \frac{1}{2}\mathcal{F}^*_{ab}\mathcal{D}_m\psi^a\sigma^m\bar{\psi}^b\\
& +g_{ab^*}F^aF^{*b} + F^a\partial_a W + F^{*a}\partial_{a^*}W^* + \frac{1}{2}g_{ab}D^aD^b + \frac{1}{2}D^a\left(\mathfrak{D}_a + 2\sqrt{2}\xi\delta^0_a\right)\\
& +(\frac{i}{4}\mathcal{F}_{abc}F^{*c} - \frac{1}{2}\partial_a\partial_b W)\psi^a\psi^b + \frac{i}{4}\mathcal{F}_{abc}F^c\lambda^a\lambda^b + \frac{1}{\sqrt{2}}(g_{ac^*}k^{*c}_b + \frac{1}{2}\mathcal{F}_{abc}D^c)\psi^a\lambda^b\\
& +(-\frac{i}{4}\mathcal{F}^*_{abc}F^c - \frac{1}{2}\partial_{a^*}\partial_{b^*}W^*)\bar{\psi}^a\bar{\psi}^b - \frac{i}{4}\mathcal{F}^*_{abc}F^{*c}\bar{\lambda}^a\bar{\lambda}^b + \frac{1}{\sqrt{2}}(g_{ca^*}k_b{}^c + \frac{1}{2}\mathcal{F}^*_{abc}D^c)\bar{\psi}^a\bar{\lambda}^b\\
& -i\frac{\sqrt{2}}{8}(\mathcal{F}_{abc}\psi^c\sigma^n\bar{\sigma}^m\lambda^a - \mathcal{F}^*_{abc}\bar{\lambda}^a\bar{\sigma}^m\sigma^n\bar{\psi}^c)v^b_{mn} - \frac{i}{8}\mathcal{F}_{abcd}\psi^c\psi^d\lambda^a\lambda^b + \frac{i}{8}\mathcal{F}^*_{abcd}\bar{\psi}^c\bar{\psi}^d\bar{\lambda}^a\bar{\lambda}^b,
\end{aligned}
$$

where $\mathcal{D}_m$ represents the gauge covariant derivative.

3. Extended Supersymmetry Transformation

Combining the $\mathcal{N}=1$ supersymmetry transformation with the $\mathfrak{R}$ transformation, we can construct the $\mathcal{N}=2$ supersymmetry transformation acting on our model:

$$
\begin{aligned}
\boldsymbol{\delta} A^a &= \sqrt{2}\boldsymbol{\eta}_j\boldsymbol{\lambda}^{ja}, \quad \boldsymbol{\delta} v^a_m = i\boldsymbol{\eta}_j\sigma_m\bar{\boldsymbol{\lambda}}^{ja} - i\boldsymbol{\lambda}_j{}^a\sigma_m\bar{\boldsymbol{\eta}}^j\\
\boldsymbol{\delta}\boldsymbol{\lambda}_j{}^a &= (\sigma^{mn}\boldsymbol{\eta}_j)v^a_{mn} + \sqrt{2}i(\sigma^m\bar{\boldsymbol{\eta}}_j)\mathcal{D}_m A^a + i(\boldsymbol{\tau}\cdot\boldsymbol{D}^a)_j{}^k\boldsymbol{\eta}_k - \frac{1}{2}\boldsymbol{\eta}_j f^a_{\ bc}A^{*b}A^c,
\end{aligned} \qquad (8)
$$

where $\boldsymbol{\lambda}_i{}^a = \binom{\lambda^a}{\psi^a}$ and $\boldsymbol{\lambda}^{ia} = \epsilon^{ij}\boldsymbol{\lambda}_j{}^a$ (and similarly for the supersymmetry parameters η_i). The $\boldsymbol{D}^a$ are auxiliary fields

$$
\boldsymbol{D}^a = \hat{\boldsymbol{D}}^a - \sqrt{2}g^{ab^*}\partial_{b^*}\left(\boldsymbol{\mathcal{E}}A^{*0} + \boldsymbol{\mathcal{M}}\mathcal{F}^*_0\right) \qquad (9)
$$

$$
\boldsymbol{\mathcal{E}} = (0,\ -e,\ \xi), \quad \boldsymbol{\mathcal{M}} = (0,\ -m,\ 0), \qquad (10)
$$

where $\hat{\boldsymbol{D}}^a$ is the fermion bilinear terms of auxiliary fields and is a real triplet under $SU(2)$. Thus, the $\mathcal{N}=2$ transformation (8) is $SU(2)$ covariant provided the two three-dimensional real vectors $\boldsymbol{\mathcal{E}}$ and $\boldsymbol{\mathcal{M}}$ transform as triplets. Their actual form (10) tells us that the rigid $SU(2)$ has been gauge fixed in this six-dimensional parameter space of $(\boldsymbol{\mathcal{E}},\ \boldsymbol{\mathcal{M}})$, by making these two vectors point to a specific direction. The manifest $SU(2)$ covariance is lost at this point.

A very important property of the triplet of the auxiliary fields $\boldsymbol{D}^a$ is that it is complex as opposed to be real. Indeed, it has a constant imaginary part:

$$
\mathrm{Im}\ \boldsymbol{D}^a = \delta^a{}_0(-\sqrt{2}m)\,(0,1,0)\,. \qquad (11)
$$

This supplies an essential ingredient for partial breaking of $\mathcal{N}=2$ supersymmetry.

4. Some Properties of the vacuum

Because $\mathcal{F}_a$ transforms in the adjoint representation of $U(N)$, we fix $\mathcal{F}$ of the form

$$
\mathcal{F} = f(A^0) + cA^0\ \mathcal{G}(\hat{B}) + \hat{\mathcal{F}}(\hat{A}), \quad \hat{A} = A^{\hat{a}}t_{\hat{a}}, \quad \hat{B} = \mathrm{Tr}(\hat{A}^2)/2c_2 \qquad (12)
$$

where the indices $\hat{a}$ are for $SU(N)$ and the constant c_2 is the quadratic Casimir. Note that the $U(1)$ part and the $SU(N)$ part have non-trivial mixings as long as $c \neq 0$.

Examining the scalar potential

$$\mathcal{V} \;=\; g^{ab}\left(\frac{1}{8}\mathfrak{D}_a\mathfrak{D}_b + \xi^2\delta_a^0\delta_b^0 + \partial_a W\partial_{b^*}W^*\right), \tag{13}$$

we find a stable minimum at $A^{\hat{a}} = 0$ and

$$f_{00} = -\frac{e}{m} \pm i\frac{\xi}{m} \tag{14}$$

which represents the unbroken $SU(N)$ phase. At this point, the $U(1)$ fermion $\frac{1}{\sqrt{2}}(\lambda^0 \mp \psi^0)$ and the $SU(N)$ fermions $\frac{1}{\sqrt{2}}(\lambda^{\hat{a}} \mp \psi^{\hat{a}})$ remain massless, while the $U(1)$ fermion $\frac{1}{\sqrt{2}}(\lambda^0 \pm \psi^0)$ and the $SU(N)$ fermions $\frac{1}{\sqrt{2}}(\lambda^{\hat{a}} \pm \psi^{\hat{a}})$ become massive with masses, $|-m\langle f_{000}\rangle|$ and $|-mc\langle\mathcal{G}'\rangle|$, respectively. Here, $\langle\cdots\rangle$ is the expectation value of $\cdots$ at the vacuum. Because

$$\left\langle\frac{\delta(\lambda^0 \mp \psi^0)}{\sqrt{2}}\right\rangle = \mp 2mi(\eta_1 \pm \eta_2)\;, \qquad \left\langle\frac{\delta(\lambda^0 \pm \psi^0)}{\sqrt{2}}\right\rangle = 0, \tag{15}$$

the $U(1)$ massless fermion is regarded as the Nambu-Goldstone fermion.

In our model the $U(1)$ fermionic shift noted in ref [4] is realized on the vacuum as an approximate symmetry coming from spontaneously broken supersymmetry.

5. $\mathcal{N} = 2$ Supercurrents

The conserved R current J associated with the $U(1)_R$ transformation can be constructed when $\mathcal{F}$ is a degree two polynomial. Though the R current is not conserved with generic $\mathcal{F}$, we can construct the conserved $\mathcal{N} = 2$ supercurrents of our model as a broken $\mathcal{N} = 2$ supermultiplet of currents [5] using the R current. The local $U(1)_R$ variation of $\mathcal{L}$ implies that

$$\partial_m\left(-\frac{1}{2}\,\mathrm{tr}\,\bar{\sigma}^m J\right) = i\left(\sum_{n,j}(n-2)\frac{\partial}{\partial h_j^{(n)}}\right)\mathcal{L} \equiv \Delta_h\mathcal{L}, \tag{16}$$

where we write $\mathcal{F}$ as $\sum_{n,j} h_j^{(n)} C_j^{(n)}(A^a)$ with $C_j^{(n)}(A^a)$ be n-th order $U(N)$ invariant polynomials in A^a, and $h_j^{(n)}$ their coefficients. Acting the supersymmetry transformation on (16), and noting that $\boldsymbol{\delta}\mathcal{L} = \partial_m X^m$ with some X^m, we obtain a general construction of the conserved $\mathcal{N} = 2$ supercurrents of our model;

$$\boldsymbol{\eta}_j\boldsymbol{\mathcal{S}}^{(j)m} + \bar{\boldsymbol{\eta}}^j\bar{\boldsymbol{\mathcal{S}}}_{(j)}^{\;m} \equiv -\frac{1}{2}\,\mathrm{tr}(\bar{\sigma}^m\boldsymbol{\delta}J) - \Delta_h X^m. \tag{17}$$

The form of the supercurrents given above tells us that our model does not permit a universal coupling to $\mathcal{N} = 2$ supergravity. The piece $-\Delta_h X^m$ is not generic and depends on the functional form of the prepotential $\mathcal{F}(A)$ in A. This supports the point of view that

$\mathcal{N}=2$ supersymmetric gauge models with nontrivial Kähler potential should be viewed as a low energy effective action. Further acting the supersymmetry transformation on (17), we can read off the constant matrix C^i_J in (2) as $C_i{}^j = +2m\xi(\boldsymbol{\tau}_1)_i{}^j$.

6. General Analysis of the Vacua with Partial Supersymmetry Breaking

Let us begin a more systematic analysis of the vacua with partial supersymmetry breaking which applies not only the unbroken but also the broken phase of $SU(N)$ gauge symmetry. Let us convert the scalar potential (13) into

$$\mathcal{V} = \frac{1}{8} g_{bc}\mathfrak{D}^b\mathfrak{D}^c + \frac{1}{2} g^{bc}\tilde{\boldsymbol{D}}_b^* \cdot \tilde{\boldsymbol{D}}_c \quad , \tag{18}$$

where

$$\tilde{\boldsymbol{D}}_b = \sqrt{2}\left(0, \partial_{b^*}W^*, -\xi\delta_b{}^0\right) \tag{19}$$

is the bosonic contribution of the auxiliary field $\boldsymbol{D}^a$ with its index lowered by g_{ba}. Noting that $\langle\mathfrak{D}^a\rangle = 0$, which follows from $\langle A^r\rangle = 0$, where the index r is for non-Cartan generators of $u(N)$, we derive the following condition on the vacuum;

$$\begin{aligned} 0 &= -4i\left\langle \partial\mathcal{V}/\partial A^a\right\rangle = \left\langle \mathcal{F}_{abc}\tilde{\boldsymbol{D}}^b \cdot \tilde{\boldsymbol{D}}^c \right\rangle \\ &= 2\left\langle \mathcal{F}_{abc} g^{bb'} g^{cc'} (\mathrm{Re}\,\partial_{b'}W)(\mathrm{Re}\,\partial_{c'}W)\right\rangle \\ &\quad +4im\left\langle \mathcal{F}_{ab0} g^{bb'}(\mathrm{Re}\,\partial_{b'}W)\right\rangle + 2\left\langle \mathcal{F}_{abc}g^{b0}g^{c0}\right\rangle \xi^2 - 2\left\langle \mathcal{F}_{a00}\right\rangle m^2. \end{aligned} \tag{20}$$

This includes the case of the vacuum with unbroken $SU(N)$ gauge symmetry as that satisfying $\langle\partial_a W\rangle = \delta_a{}^0(e + m\langle f_{00}\rangle)$, and $\langle g^{\hat{b}0}\rangle = 0$.

Let us now turn to analyze the condition of partial breaking of $\mathcal{N}=2$ supersymmetry. We have

$$\left\langle \boldsymbol{\delta\lambda}_j{}^a\right\rangle = i\left\langle (\boldsymbol{\tau}\cdot\tilde{\boldsymbol{D}}^a)_j{}^k \boldsymbol{\eta}_k\right\rangle . \tag{21}$$

In order to find a Nambu-Goldstone fermion from the $\mathcal{N}=2$ doublets of $U(N)$ fermions, we must find a set of nonvanishing coefficients C_a such that

$$\begin{aligned} \tilde{\boldsymbol{D}}(C_a) &\equiv \sum_a C_a \tilde{\boldsymbol{D}}^a, \\ 0 &= \left\langle \det \boldsymbol{\tau}\cdot\tilde{\boldsymbol{D}}\right\rangle = \left\langle \tilde{\boldsymbol{D}}\cdot\tilde{\boldsymbol{D}}\right\rangle . \end{aligned} \tag{22}$$

It is straightforward to spell out (22) in terms of the real and imaginary parts of $\tilde{\boldsymbol{D}}_a$, using (19).

Let us further assume

$$\langle \mathrm{Re}\,\partial_b W\rangle = 0 \ . \tag{23}$$

Eq.(20) then gives us (for non-zero $\langle \mathcal{F}_{a00} \rangle$)

$$\frac{\langle \mathcal{F}_{abc} \rangle \langle g^{b0} \rangle \langle g^{c0} \rangle}{\langle \mathcal{F}_{a00} \rangle} = \frac{m^2}{\xi^2} \qquad (\text{no sum on } \ a). \tag{24}$$

On the other hand, the condition (22) gives us

$$\left(\sum_b \frac{C_b}{C_0} \langle g^{b0} \rangle \right)^2 = \frac{m^2}{\xi^2} \ . \tag{25}$$

Eq.(23),(24) and the existence of the coefficients C_b satisfying (25) are the conditions that the vacuum state satisfies and include the vacuum discussed in §4.

7. Acknowledgements

This work is supported in part by the Grant-in-Aid for Scientific Research(16540262) from the Ministry of Education, Science and Culture, Japan, and by the 21 century COE program "Constitution of wide-angle mathematical basis focused on knots".

8. References

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